Should Exchange Rates be Ignored in the Setting of Monetary Policy?*

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Abstract

The aim of this paper is to highlight the role of parameter uncertainty in optimal open-economy monetary policy. Parameter uncertainty is introduced by assuming that the coefficients of the model are random, but with known stochastic properties. The results indicate that the Taylor rule is a reasonable approximation to optimal monetary policy under uncertainty, even in an open economy. The intuition behind the result is the "Brainard conservatism principle," which drives the response coefficients of the exchange rate variables in the optimal rule close to zero, while the response coefficients on inflation and output are similar to values suggested by Taylor. The model is estimated on German data and the Taylor rule is found to be roughly similar to the optimal rule under uncertainty.

Keywords: Model uncertainty, open-economy inflation targeting, Taylor rules

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1 Introduction

A popular benchmark for monetary policy in closed economies is the Taylor (1993) rule. The Taylor rule specifies that the instrument of the central bank, usually the short-term interest rate, should be a linear function of current inflation and output gap.[1] The popularity of the Taylor rule stems both from its simplicity and its ability to explain actual Federal Reserve interest rate behavior. In addition, the Taylor rule also possesses many attractive features which make it close to optimal policy in microfounded closed-economy models (see Woodford (2001)).

In an open economy, however, the real exchange rate can on theoretical grounds be expected to be an important indicator for monetary policy: (1) the real exchange rate affects the aggregate demand in an economy by expenditure switching between exports and imports; and (2) the real exchange rate has a direct effect on the prices of imported final and intermediate goods. Therefore, it is reasonable to expect that a Taylor rule augmented with the real exchange rate could lead to a better macroeconomic performance in terms of stabilizing output and inflation in an open economy.[2]

Indeed, several recent studies find that macroeconomic performance is improved by considering monetary policy rules that include the real exchange rate (see, for example, Ball (1999), Svensson (2000), Dennis (2000), Batini *et al.* (2001)). As the determination of exchange rates, as well as the real exchange rate's effect on the economy, are far from well understood in practice, policy makers face a dilemma in responding to the recommendations coming out of these studies. While a well-oiled theoretical apparatus exists for analyzing exchange rate movements, such as purchasing power parity (PPP) and uncovered interest parity (UIP), there is an absence of reliable empirical relationships (see Hodrick (1987), Eichenbaum and Evans (1995), Goldberg and Knetter (1997), and Chinn and Fujii (2001)). Furthermore, there is substantial disagreement in the literature about the modeling of exchange rates as following rational expectations or more traditional adaptive expectations (see Frankel and Froot (1987) and Eichenbaum and Evans (1995)). Therefore, it is here argued that the exchange rate aspects of the monetary

[1] Based on the U.S. macroeconomic experience between 1987-1992, Taylor argued that the response coefficient to inflation should be 1.5, while the response coefficient on output should be 0.5. Thus, according to the rule, higher inflation and higher output imply a higher interest rate.

[2] Taylor (2001, footnote 3) writes: "The multicountry model that I used in my 1993 paper...includes a big role for the exchange rate. Simulations of that model and other similar models showed, however, that if the central bank reacted strongly to the exchange rate then the macroeconomic performance would worsen. That was why I omitted the exchange rate in the 1993 rule for the Fed. But it is not clear that the same conclusion would hold for other countries."

transmission mechanism are clouded in relatively large uncertainties.

The focus of this paper is to analyze the effects of introducing parameter uncertainty in an open-economy model of the monetary transmission mechanism. The strategy throughout the paper is to compare the benchmark case of optimal policy when parameters are constant (certainty equivalence), to optimal policy when parameters are assumed to be random. The vehicle for this analysis is an open-economy model due to Ball (1999), in which a result is that an optimal monetary policy rule should indeed include the real exchange rate. The methodology for solving the model under parameter uncertainty is the same as Söderström (2002) used in a closed-economy analysis.

Previewing the results of this paper, compared to optimal responses with constant parameters, parameter uncertainty causes the optimal response coefficients for inflation, output gap and the real exchange rate to become more muted. In fact, with realistic parameter uncertainty, the optimal response coefficients on annual inflation and output gap are close to the values suggested by the Taylor rule, while the response coefficents on the real exchange rate are near zero. The driving mechanism of this result is "Brainard's conservatism principle," which implies that policy makers are optimally more cautious when facing parameter uncertainty.[3] Therefore, in this case, the Taylor rule provides a reasonable approximation to optimal monetary policy – even in an open economy. Furthermore, the results indicate that uncertainty about the effects of exchange rate movements can explain why central banks empirically do not seem to respond to the exchange rate explicitly (see Taylor (2001) and the evidence cited therein). The model is also estimated on annual German data, 1975-1998, and the Taylor rule is found to be close to the optimal rule under uncertainty.

The paper is structured as follows. Section 2 contains a background discussion of previous research. Section 3 outlines the baseline model, introduces parameter uncertainty and the central bank loss function. Section 4 numerically compares the optimal policy response in the presence of parameter uncertainty against the benchmark of no parameter uncertainty. The model is estimated in Section 5 and Section 6 concludes. The Appendix contains technical details.

[3]For further discussion of the Brainard conservatism principle, see Brainard (1967) and Blinder (1998). Also, Söderström (2002) analyzes a case where the principle does not apply.

2 Background and Previous Research

Monetary policy in open economies is currently an active field of research.[4] Despite voluminous research, there is no consensus on what the role of the exchange rate should be in the formulation of monetary policy. Several answers are possible. One possibility is to use a monetary policy rule that ignores the exchange rate and only focuses on inflation and output gap, i.e. following a Taylor rule. Based on economic theory, many would object to this solution. For example, Obstfeld and Rogoff (1995, p. 93) write:

> Clearly the answer is no, since the exchange rate – like the interest rate – is an important economic indicator. The interpretation of exchange rate movements is complex, however, and not just because short-term volatility often clouds the role of underlying fundamentals. ...[I]t is important to recognize that even in a world with flexible prices there can be substantial departures from PPP, in the short run and even over decades.

A second possibility, again suggested by Obstfeld and Rogoff (1995), is to follow a rule-of-thumb in regards to the level of the real exchange rate – relaxing monetary policy in response to a real exchange rate appreciation. A third option is to consider a monetary policy rule that reacts to the percentage change in the real exchange rate from one period to the next.

Several recent papers such as Ball (1999), Svensson (2000), Batini *et al.* (2001) and Taylor (1999, 2001), consider interest rate rules of the form

$$i_t = f_\pi \pi_t + f_y y_t + f_q q_t + f_{q-1} q_{t-1}, \tag{1}$$

where i_t is the short-term interest rate, interpreted as the instrument of the central bank; π_t denotes inflation; y_t is the real output gap; and q_t represents the real exchange rate, where an increase is an appreciation.[5]

In the context of equation (1), the real exchange rate's role in a monetary policy rule can be formalized by focusing on the coefficients, $\{f_\pi, f_y, f_q, f_{q-1}\}$.

[4] A website dedicated to collecting recent research on monetary policy rules in open economies can be found at http://www.geocities.com/monetaryrules/mpoe.htm.

[5] All variables are measured as deviations from average values.

If $f_\pi > 1$ and $f_y > 0$, while $f_q = f_{q-1} = 0$, then the central bank only responds to current inflation and output gap, with no reaction to the exchange rate. The monetary policy rule is then similar to the Taylor rule.

If the central bank follows the rule-of-thumb behavior proposed by Obstfeld and Rogoff (1995), then $f_q < 0$, and $f_{q-1} = 0$. An appreciation of the real exchange rate (from its average level) would then lead the central bank to lower the short-term interest rate.

If $|f_q| = |f_{q-1}| > 0$, the central bank reacts to the percentage change in the real exchange rate.

Allowing for response to a lagged exchange rate in the monetary policy rule could imply somewhat more complicated dynamics. Following the reasoning in Taylor (2001), f_q and f_{q-1} could be of different signs. Indeed, this is what Ball (1999) finds in his study of the monetary transmission in an open economy, where $f_q = -0.37$ while $f_{q-1} = 0.17$. Ball's monetary policy rule implies that an appreciation of the exchange rate of ten percent would require a cut in the interest rate by 3.7 percent, to be followed in the next period by a partial increase of 1.7 percent. In a two-period perspective, there is then a two-percent reduction in the interest rate in response to the appreciation.

In a study with more elaborate microfoundations, Svensson (2000) uses a model with forward-looking agents where a monetary policy rule considered has the following exchange rate reaction coefficients: $f_q = -0.45$, $f_{q-1} = 0.45$. Svensson's rule thus considers the percentage change in the real exchange rate. Using this rule, Svensson finds conflicting results: inflation variability decreased, but at the cost of increased output variability.

In an application to the Euro-area, Taylor (1999) applies a monetary policy rule as in equation (1), with exchange rate reaction coefficients of $f_q = -0.25$, $f_{q-1} = 0.15$. Simulating this policy rule, Taylor found that the rule lead to better performance for France and Italy, but worse for Germany.

The literature discussed above assumes that the model is known with certainty. While there is a vast literature on monetary policy under uncertainty, most research is conducted in closed-economy models (see, e. g., Söderström (2002), Sack (2000), Hansen and Sargent (2000), Giannoni (2000), and Wieland (1998)). There are a few exceptions, however. Sargent (1998) applies robust control techniques to analyze the effects of dynamic shock uncertainty in the context of Ball's (1999) model. Srour (1999) also uses Ball's model to analytically investigate exchange rate uncertainty in a "strict" inflation targeting framework, i.e. where inflation is the only argument in the loss function, but is forced to assume that there is no exchange rate pass-through to inflation, in order to solve it analytically. In a two-period model, Srour (1999) finds that uncertainty about the exchange rate coefficient in the aggregate demand equation makes policy makers more cautious in line with the "Brainard conservatism principle." In this paper, in contrast to Srour (1999), the goal function of the central bank includes both inflation and output stabilization objectives, the horizon is infinite, and there is exchange rate pass-through to inflation. Furthermore, the framework used in this paper allows for multiplicative uncertainty in all parameters, as in Söderström (2002).

In a study with a closely related objective, Leitemo and Söderström (2001) examine the performance of simple monetary policy rules in a forward-looking, open-economy framework. While the question they ask is similar in spirit to the issue considered in this paper, their methodology is very different. Introducing parameter uncertainty in a forward-looking model is a daunting technical task. Therefore, they do not consider the effects of introducing parameter uncertainty, but focus on the performance of various simple rules under different assumptions of the determination of exchange rates, persistence of risk premiums, degree of rationality in exchange rate expectations, and exchange rate pass-through to inflation. The results of Leitemo and Söderström (2001) indicate that optimized Taylor rules are robust; adding a response to the exchange rate in the simple rule adds only marginal improvements in terms of economic stability.

A backward-looking model, as formulated by Ball (1999), is here used due to its tractability and ability to capture the salient features of data. A forward-looking model would of course be theoretically more elegant, but the mathematical complexity of solving a forward-looking model with parameter uncertainty is prohibitive. Also, the relevance of using rational expectations models for actual policy evaluation in a stable macroeconomic environment is not established. Evidence on this issue is provided by Estrella and Fuhrer (1999), who find that backward-looking specifications fit the data better than their forward-looking counterparts.

It is important to emphasize that the focus of this paper is somewhat different than the dominant paradigm in research on monetary policy rules. Most papers in the monetary policy rules literature start with the specification of a monetary policy rule and then solve the macroeconomic model (usually numerically). The stochastic properties of the target variables, inflation and the output gap, are then evaluated for each rule in question and the best rule is selected.

Instead of evaluating the stochastic properties of the target variables, this paper emphasizes the properties of the optimal response coefficients, $\{f_\pi, f_y, f_q, f_{q-1}\}$, in equation (1). There is thus no effort to evaluate the welfare properties of various rules, but rather to evaluate the robustness of optimal open-economy rules to parameter uncertainty. The model set-up considered here is structured so that the optimal monetary policy rule will correspond to the rule in (1). Attention will then focus on how sensitive the optimal rule in equation (1) is to various assumptions about the uncertainty facing the central bank.

3 A Baseline Model

The model is adapted from Ball (1999), and can be described by three equations: (1) an open economy backward-looking Phillips curve; (2) an open economy backward-looking aggregate demand curve; and (3) an interest parity equation relating the real exchange rate and real interest rate

$$\pi_{t+1} = \alpha_\pi \pi_t + \alpha_y y_t - \alpha_q (q_t - q_{t-1}) + \epsilon_{t+1} \quad (2)$$

$$y_{t+1} = \beta_y y_t - \beta_r (i_t - \pi_t) - \beta_q q_t + \eta_{t+1} \quad (3)$$

$$q_{t+1} = \gamma_r (i_t - \pi_t) + \nu_{t+1}, \quad (4)$$

where π_t is inflation; y_t is the log of real output; q_t is the log of real exchange rate (a higher q_t means an appreciation); and i_t is the nominal interest rate, controlled by the central bank. All variables are measured as deviations from average levels. The shock terms, ϵ_{t+1}, η_{t+1}, ν_{t+1}, are i.i.d. structural shocks with zero mean and variances σ^2_ϵ, σ^2_η, and σ^2_ν, respectively.

Inflation depends on its own lag, lagged output, the percentage change in the real exchange rate, and a shock. Equation (2) is derived by assuming that domestic goods inflation, π^d_t, is determined by

$$\pi^d_t = \pi_{t-1} + \theta y_{t-1} + \zeta_t, \quad (5)$$

which is similar to a closed-economy Phillips curve. The error term, ζ_t, is an i.i.d. zero mean shock term with known variance.

Following Ball (1999), foreign firms desire constant real prices in their home currency, from which it follows that their desired local currency real prices are $-q$. The foreign firms are assumed to adjust their prices to changes in q with a one-period lag. Analogous to domestic firms, the foreign firms also adjust their prices to lagged inflation. Import inflation, π_t^{IM}, is therefore given by

$$\pi_t^{IM} = \pi_{t-1} - (q_{t-1} - q_{t-2}). \tag{6}$$

Aggregate inflation in equation (2) is a weighted average of (5) and (6), based on the share of imports in the price index. If the share of imports in an economy is ω, then the aggregate inflation is given by:

$$\begin{aligned} \pi_t &= \omega\pi_t^{IM} + (1-\omega)\pi_t^d, \\ &= \omega\left[\pi_{t-1} - (q_{t-1} - q_{t-2})\right] + (1-\omega)\left[\pi_{t-1} + \theta y_{t-1} + \zeta_t\right] \\ &= \pi_{t-1} + (1-\omega)\theta y_{t-1} - \omega(q_{t-1} - q_{t-2}) + (1-\omega)\zeta_t, \end{aligned}$$

from which equation (2), with coefficients $\alpha_y = (1-\omega)\theta$, $\alpha_q = \omega$, and $\epsilon_t = (1-\omega)\zeta_t$, is derived.

Output in equation (3) depends on its own lag, on lags of the real interest rate, the log of the real exchange rate, and a shock. The real exchange rate affects aggregate demand due to its influence on expenditure switching between exports and imports; an appreciation makes imports cheaper while exports become more expensive, thus affecting aggregate demand adversely.

The final equation (4) of the model posits a link between the real interest rate and the real exchange rate. It reflects the idea that a rise in the interest rate makes domestic assets more attractive, leading to an appreciation. The shock term, v_{t+1}, captures other influences on the real exchange rate such as expectations, investor confidence and foreign interest rates.

One change is made to Ball's (1999) original model formulation: a lead is introduced in the interest parity relation, (4). Ball's formulation is $q_t = \gamma_r r_t + v_t$, where r_t denotes the real interest rate. The change is made to ensure that the optimal rule will be of the same form as equation (1) discussed in Section 2.

Empirically, there is little evidence of a contemporaneus impact of contractionary monetary policy on exchange rates. (see, for example, Bagliano *et al.* (2001)).[6] Further motivation comes from Eichenbaum and Evans (1995), who estimate the effect of monetary policy on nominal and real exchange rates. They find that a contractionary monetary shock leads to persistent

[6]Bagliano *et al.* (2001, p. 16) write: "Our analysis shows that there is no within-month simultaneous feedback between policy rates and the exchange rate....The results from simultaneous feedbacks, impulse response and variance decompositions reveal that monetary factors play a very limited direct role in explaining exchange rate fluctuations."

appreciation of the exchange rate and significant departures from the uncovered interest parity; they find no evidence of a contemporaneous impact of contractionary monetary policy on exchange rates. Instead, the maximal impact of contractionary monetary policy in six countries lie between 22-33 months! Based on this evidence, the change made to Ball's original formulation may make sense.

The timing of the transmission of monetary policy is then as follows. A contractionary monetary policy affects the output gap, y, and the level of exchange rate, q, in period $t+1$. The effect of the rise in the interest rate reaches inflation, π, in period $t+2$.

3.1 Baseline Model with Parameter Uncertainty

The baseline model assumes that the coefficients of the explanatory variables are known with certainty. Uncertainty about the coefficient of a variable in the transmission mechanism implies that the larger the change in the variable, the larger the uncertainty about its effect on the economy. The conventional wisdom, following Brainard's (1967) result, is that uncertainty induces caution with the policy maker in an attempt to minimize the deviations of the variable in question.

With parameter uncertainty, the baseline model will take the following form

$$\begin{aligned}
\pi_{t+1} &= \alpha_{t+1,\pi}\pi_t + \alpha_{t+1,y}y_t - \alpha_{t+1,q}\left(q_t - q_{t-1}\right) + \varepsilon_{t+1} && (7)\\
y_{t+1} &= \beta_{t+1,y}y_t - \beta_{t+1,r}(i_t - \pi_t) - \beta_{t+1,q}q_t + \eta_{t+1} && (8)\\
q_{t+1} &= \gamma_{t+1,r}(i_t - \pi_t) + \nu_{t+1}. && (9)
\end{aligned}$$

Assume that the coefficients follow a known stochastic process

$$\kappa_{t+1} = \kappa + v_{t+1}^{\kappa}, \tag{10}$$

where $\kappa = \left\{\alpha_\pi, \alpha_y, \alpha_y, \beta_y, \beta_r, \beta_q, \gamma_r\right\}$. The shock term is i.i.d with zero mean and known variance. The shock terms are independent of each other and the structural shocks. In each period, the parameter shocks are drawn from the same distribution (thus there is no learning).

3.2 Loss function

The central bank is assumed to derive the optimal path for the interest rate subject to the development of the economy. The central bank's intertemporal loss function is quadratic in inflation and output

$$\min_{\{i_{t+\tau}\}_{\tau=0}^{\infty}} E_t \sum_{\tau=0}^{\infty} \beta^{\tau} L(y_{t+\tau}, \pi_{t+\tau}), \qquad 0 < \beta < 1$$

subject to (2)-(4), or (7)-(9), depending on the assumption of constant or uncertain parameters. The period loss function is $L(y_t, \pi_t) = \pi_t^2 + \lambda y_t^2$ and β is the central bank's discount factor.

3.3 Solving for Optimal Policy

When the parameters are stochastic, finding a closed-form solution under "flexible" inflation targeting, with positive weights on both inflation and output stabilization, is impossible for the model used in this paper. Therefore, the focus will be on numerical solutions, following the approach in Söderström (2002).

In matrix form, the model in (7) - (9) can be expressed as

$$\begin{bmatrix} \pi_{t+1} \\ y_{t+1} \\ q_t \\ q_{t+1} \end{bmatrix} = \begin{bmatrix} \alpha_{\pi,t+1} & \alpha_{y,t+1} & \alpha_{q,t+1} & -\alpha_{q,t+1} \\ \beta_{r,t+1} & \beta_{y,t+1} & 0 & -\beta_{q,t+1} \\ 0 & 0 & 0 & 1 \\ -\gamma_{r,t+1} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \pi_t \\ y_t \\ q_{t-1} \\ q_t \end{bmatrix} + \tag{11}$$

$$+ \begin{bmatrix} 0 \\ -\beta_{r,t+1} \\ 0 \\ \gamma_{r,t+1} \end{bmatrix} i_t + \begin{bmatrix} \epsilon_{t+1} \\ \eta_{t+1} \\ 0 \\ \nu_{t+1} \end{bmatrix}.$$

To analyze the central bank's optimization problem, (11) is written in state-space form as

$$x_{t+1} = A_{t+1} x_t + B_{t+1} i_t + \varepsilon_{t+1}, \tag{12}$$

where $x_{t+1} = \begin{bmatrix} \pi_{t+1} & y_{t+1} & q_t & q_{t+1} \end{bmatrix}'$ is a state vector, and $\varepsilon_{t+1} = [\epsilon_{t+1} \ \eta_{t+1} \ 0 \ v_{t+1}]'$ is a vector of structural shocks.

The Bellman equation expressing the central bank's optimization problem can be formulated as

$$v(x_t) = \min_{i_t} \left[x_t' Q x_t + \beta E_t v\left(x_{t+1}\right) \right], \tag{13}$$

where Q is a (4×4) preference matrix of the central bank with the vector $[1, \lambda, 0, 0]$ on the diagonal, and zeros elsewhere. With a quadratic loss function and linear constraints, the value function takes the form (see Ljungqvist and Sargent (2000))

$$v\left(x_{t+1}\right) = x_{t+1}' V x_{t+1} + d, \tag{14}$$

where the matrix V is to be determined. Substituting (14) and (12) into (13) gives

$$v(x_t) = \min_{i_t} \left[x_t' Q x_t + \beta E_t \left(\left(A_{t+1} x_t + B_{t+1} i_t + \varepsilon_{t+1} \right)' V \left(A_{t+1} x_t + B_{t+1} i_t + \varepsilon_{t+1} \right) + d \right) \right]. \tag{15}$$

With constant parameters, $A_{t+1} = A$, and $B_{t+1} = B$, then certainty equivalence applies and optimal policy is independent of any uncertainty. Under certainty equivalence (CE), the optimal interest rate, i_t, is set as

$$\begin{aligned} i_t &= -\left[\left(B' \left(V + V' \right) B \right)^{-1} B' \left(V + V' \right) A \right] x_t, \\ i_t &= F^{CE} x_t, \\ i_t &= f_\pi^{CE} \pi_t + f_y^{CE} y_t + f_{q-1}^{CE} q_{t-1} + f_q^{CE} q_t, \end{aligned} \tag{16}$$

which is of the same form as equation (1), discussed in Section 2.

As is well known, with stochastic parameters certainty equivalence does not apply, and optimal policy will depend on the coefficient variances. It is shown in the Appendix that the extra terms to take into account due to parameter uncertainty in (15) are

$$\begin{aligned} v(x_t) &= \min_{i_t} [x_t' Q x_t + \beta \left(A x_t + B i_t \right)' V \left(A x_t + B i_t \right) + \\ & v_{11} \left(x_t' \Sigma_{11}^A x_t + \Sigma_\epsilon \right) + v_{22} \left(x_t' \Sigma_{22}^A x_t + 2 x_t' \Sigma_{22}^{AB} i_t + i_t' \Sigma_{22}^B i_t + \Sigma_\eta \right) \\ & v_{44} \left(x_t' \Sigma_{44}^A x_t + 2 x_t' \Sigma_{44}^{AB} i_t + i_t' \Sigma_{44}^B i_t + \Sigma_\upsilon \right) + d], \end{aligned}$$

where v_{ii} are the diagonal elements of V, $i = 1, 2, 3, 4$, and

$$
\begin{aligned}
\Sigma_{11}^{A} &= \begin{bmatrix} \sigma_{\alpha_\pi}^2 & 0 & 0 & 0 \\ 0 & \sigma_{\alpha_y}^2 & 0 & 0 \\ 0 & 0 & \sigma_{\alpha_q}^2 & -\sigma_{\alpha_q}^2 \\ 0 & 0 & -\sigma_{\alpha_q}^2 & \sigma_{\alpha_q}^2 \end{bmatrix}, \Sigma_{22}^{A} = \begin{bmatrix} \sigma_{\beta_r}^2 & 0 & 0 & 0 \\ 0 & \sigma_{\beta_y}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{\beta_q}^2 \end{bmatrix} \\
\Sigma_{22}^{AB} &= \begin{bmatrix} \sigma_{\beta_r}^2 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \Sigma_{44}^{AB} = \begin{bmatrix} \sigma_{\gamma_r}^2 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \Sigma_{44}^{A} = \begin{bmatrix} \sigma_{\gamma_r}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\
\Sigma_{22}^{B} &= \sigma_{\beta_r}^2, \Sigma_{44}^{B} = \sigma_{\gamma_r}^2,
\end{aligned}
$$

and $\Sigma_\epsilon, \Sigma_\eta$, and Σ_υ are the variances of the structural shock-terms. The Appendix also shows that the optimal decision rule for the central bank under parameter uncertainty (PU) is a linear function of the state vector,

$$
i_t = F^{PU} x_t,
$$

where

$$
\begin{aligned}
F^{PU} &= -\left[B'\left(V+V'\right)B + 2v_{22}\Sigma_{22}^{B} + 2v_{44}\Sigma_{44}^{B}\right]^{-1} \times \\
&\quad \left[B'\left(V+V'\right)A + 2v_{22}\Sigma_{22}^{AB} + 2v_{44}\Sigma_{44}^{AB}\right].
\end{aligned}
$$

The 1×4 vector F^{PU} thus consists of the optimal response coefficients under parameter uncertainty. The optimal interest rule is then

$$
i_t = f_\pi^{PU}\pi_t + f_y^{PU} y_t + f_{q-1}^{PU} q_{t-1} + f_q^{PU} q_t, \tag{17}
$$

the same form as in equation (1), considered in Taylor (2001), Ball (1999), and Svensson (2000), among others. Finally, the matrix V is obtained by iterating on the Ricatti equation

$$
\begin{aligned}
V &= [Q + \beta\{(A+Bf)' V (A+Bf) + v_{11}\Sigma_{11}^{A} + \\
&\quad v_{22}\left(\Sigma_{22}^{A} + 2\Sigma_{22}^{AB} f + f'\Sigma_{22}^{B} f\right) + v_{44}\left(\Sigma_{44}^{A} + 2\Sigma_{44}^{AB} f + f'\Sigma_{44}^{B} f\right)\}].
\end{aligned}
$$

4 Optimal Policy and the Effects of Exchange Rate Parameter Uncertainty

The focus of this section is to numerically evaluate how the optimal reaction function under certainty equivalence changes in response to parameter uncertainty, i.e. comparing the response coefficients, $\{f_\pi^{CE}, f_y^{CE}, f_{q-1}^{CE}, f_q^{CE}\}$ in (16) to the response coefficients $\{f_\pi^{PU}, f_y^{PU}, f_{q-1}^{PU}, f_q^{PU}\}$ under uncertainty in (17). The mean parameter values in Table 1 are taken from Ball (1999).[7]

Table 1: Parameter Values and Five Uncertainty Scenarios

Stochastic Parameters		Uncertainty Scenario (Variances)				Non-Stochastic	
	Mean	1	2	3	4		Value
$\alpha_{t+1,\pi}$	1	{0.00	0.00	0.00	0.25}	β	0.95
$\alpha_{t+1,y}$	0.4	{0.00	0.00	0.00	0.04}	λ	0.5
$\alpha_{t+1,q}$	0.2	{0.01	0.00	0.00	0.01}		
$\beta_{t+1,y}$	0.8	{0.00	0.00	0.00	0.16}		
$\beta_{t+1,r}$	0.6	{0.00	0.00	0.00	0.09}		
$\beta_{t+1,q}$	0.2	{0.00	0.01	0.00	0.01}		
$\gamma_{t+1,r}$	2	{0.00	0.00	1	1}		

Four uncertainty scenarios are introduced in Table 1, which are intended to be read in column-wise fashion. In the first uncertainty scenario (column 1), only the effect of uncertainty about the exchange rate pass-through to inflation, α_q, is considered. In the second column of the uncertainty scenarios, uncertainty about the real exchange rate's effect on aggregate demand, β_q, is introduced. In the third column of the uncertainty scenarios, the coefficient in the interest parity relation, γ_r, is assumed to be random. Finally, in the fourth column, all parameters are uncertain.

The uncertainty, in variance terms, assumed for each parameter varies in magnitude. The guiding principle in selecting the uncertainty is that each coefficient should have the same t-value (coefficient/standard error), i.e. estimated with the same level of significance. It is assumed that every coefficient is estimated with a t-value of 2, making each roughly significant. This of course translates to different variance terms. How realistic the assumed variances are will be discussed below in Section 5. The weight on output gap stabilization, λ, is set to 0.5, a value often assumed in the literature on inflation targeting.

[7] Ball (1999) selected the coefficient values to be broadly consistent with empirical models used at Bank of Canada and Reserve Bank of New Zealand.

The results of comparing the optimal response coefficients of equations (16) and (17) are reported in Table 2.

Table 2: Optimal responses under various forms of uncertainty ($\lambda = 0.5$)

	f_π	f_y	f_{q-1}	f_q
Certainty Equivalence	2.11	1.24	0.22	-0.20
Uncertainty Scenario 1: α_q uncertain	1.82	0.94	0.16	-0.11
	-14%	-24%	-27%	-45%
Uncertainty Scenario 2: β_q uncertain	2.05	1.19	0.21	-0.19
	-3%	-4%	-5%	-5%
Uncertainty Scenario 3: γ_q uncertain	2.04	1.17	0.21	-0.19
	-3%	-6%	-5%	-5%
Uncertainty Scenario 4: All uncertain	1.70	0.81	0.14	-0.10
	-19%	-35%	-45%	-50%

A quick glance at Table 2 reveals that uncertainty causes the optimal response coefficients to become more muted compared to certainty equivalence. This observation amounts to the celebrated "Brainard conservatism principle": parameter uncertainty results in less aggressive interest-rate setting by the central bank.

A closer examination of the optimal responses under certainty equivalence shows that, compared to the coefficients suggested in Taylor's rule, the reponses to inflation, f_π^{CE}, and output, f_y^{CE}, are quite aggressive. Furthermore, the optimal responses to the current and lagged exchange rates, f_q^{CE} and f_{q-1}^{CE}, are negative and positive, respectively. This accords well with Ball's (1999) finding.

Row number two in Table 2 corresponds to uncertainty scenario number 1 and describes the optimal response coefficients to uncertainty in the pass-through to inflation, α_q. When parameter uncertainty is introduced in the α_q coefficient, the optimal response coefficient to inflation, f_π^{PU}, drops from 2.11 to 1.82, a drop of 14%; the optimal response coefficient to the output gap, f_y^{CE}, drops 24 %; the optimal response coefficient to the lagged exchange rate, f_{q-1}^{PU}, drops 27%; and the optimal response coefficient to the current real exchange rate, f_q^{PU}, drops 45%.[8]

[8]For reasons that will be discussed in the next subsection, the percentage change in the optimal response coefficient on inflation is understated, while the percentage change in the optimal response coefficient on the current real exchange rate is overstated.

The effects of introducing uncertainty in the β_q coefficient are analyzed in uncertainty scenario number two. The optimal response coefficients are less sensitive to uncertainty in the β_q compared to uncertainty in the pass-through to inflation, α_q: the uncertainty reduces the optimal response coefficients by 3-5%. Similarly, uncertainty in γ_q reduces the optimal response coefficients by only 3-6%.

When uncertainty is introduced in all coefficients, the resulting drop compared to certainty equivalence is more marked. Uncertainty reduces the optimal response coefficient to the current exchange rate by 50%, and the lagged exchange rate by 45%. The optimal response coefficients for inflation, f_{π}^{PU}, and the output gap, f_y, are now closer to the values that Taylor suggested, $f_{\pi} = 1.5$ and $f_y = 0.5$.

To sum up the results in this section, the sensitivity of the optimal response coefficients varies with which coefficient is assumed to be uncertain. Optimal responses are quite sensitive to uncertainty in the exchange rate pass-through to inflation. Similarly, optimal responses are sensitive when all coefficients are uncertain. In contrast, uncertainty about γ_r, the coefficient in the interest parity relation, does not appear to have much on an effect of optimal policy under uncertainty.

4.1 Effects of Increasing the Uncertainty

Given the imprecise knowledge of how the exchange rate is determined and how exchange rate variables affect the economy, a reasonable assumption might be that the exchange rate parameters are estimated with larger uncertainty than other parameters. (For more evidence on this, see Section 5.) To analyze the effect on optimal monetary policy of large variances in the exchange rate parameters, the uncertainty is incrementally increased. In this subsection, the uncertainty (variance) in the exchange rate parameters is increased from 0 to 50 in increments of 0.25. Again, the weight on the output gap is assumed to be $\lambda = 0.5$.

The results are displayed in Figures 1-3. On the $x-$axis, the variance is shown. The optimal response under certainty, here represented by the solid line, is of course independent of the amount of uncertainty, and therefore constant as the variance increases. The optimal responses under uncertainty are represented by a dotted line. In the top-left panel, the effects of the increased uncertainty on the optimal response coefficients to inflation are depicted, f_{π}^{CE} and f_{π}^{PU}; in the top-right panel the optimal responses to the output gap, f_y^{CE} and f_y^{PU}; in the bottom-left panel, the optimal response to the lagged exchange rate, f_{q-1}^{CE} and f_{q-1}^{PU}; and in the

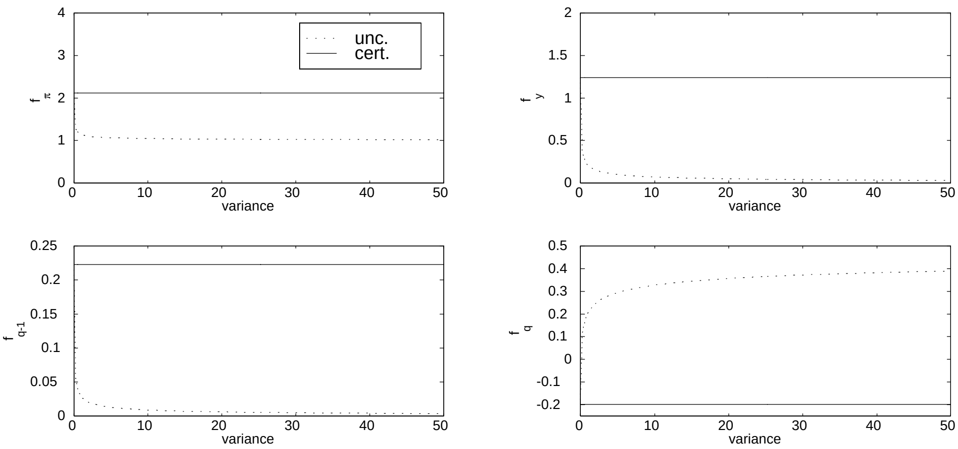

Figure 1: Effects of increasing α_π uncertainty. The top-left panel is the optimal response to inflation as uncertainty increases. The top-right panel depicts the optimal response of output; the bottom-left panel shows the response of the lagged real exchange rate; and, finally, the bottom-right panel depicts the optimal response of the current real exchange rate.

bottom-right panel, the optimal response to the current exchange rate, f_q^{CE} and f_q^{PU}.

In Figure 1, the effects of increasing the uncertainty associated with α_q, are displayed. The response coefficient of inflation under uncertainty, f_π^{PU}, declines rapidly as uncertainty increases only marginally, and then settles "asymptotically" to 1; the response coefficient of output under uncertainty, f_y^{PU}, initially becomes more muted as uncertainty increases before leveling off close to zero; the response coefficient of the lagged real exchange rate, f_{q-1}^{PU}, decreases quickly with only moderate amount of uncertainty before approaching zero as uncertainty increases. The response to the current exchange rate, f_q^{PU}, quickly approaches zero as uncertainty increases, but it does not level out at zero. Instead, with increased uncertainty, the responses become positive, before settling down around 0.5. This is puzzling at first blush as Brainard's reasoning implies that as uncertainty increases, the optimal response is to do nothing. What mathematically drives the result in the bottom-right panel of Figure 1 is the non-diagonal covariance terms in Σ_{11}^A, discussed in Section 3, which comes from the fact that the coefficient, $\alpha_{t+1,q}$, on q_t and q_{t-1} is the same in equation (7). Therefore, Brainard's result that it is optimal to do nothing in the limit as uncertainty increases is not obtained in this case. Economically, the switching of signs of the response coefficient is harder to explain. Nevertheless, the main point is that the response coefficient on f_q^{PU} is sensitive to uncertainty, which casues the response coefficient to

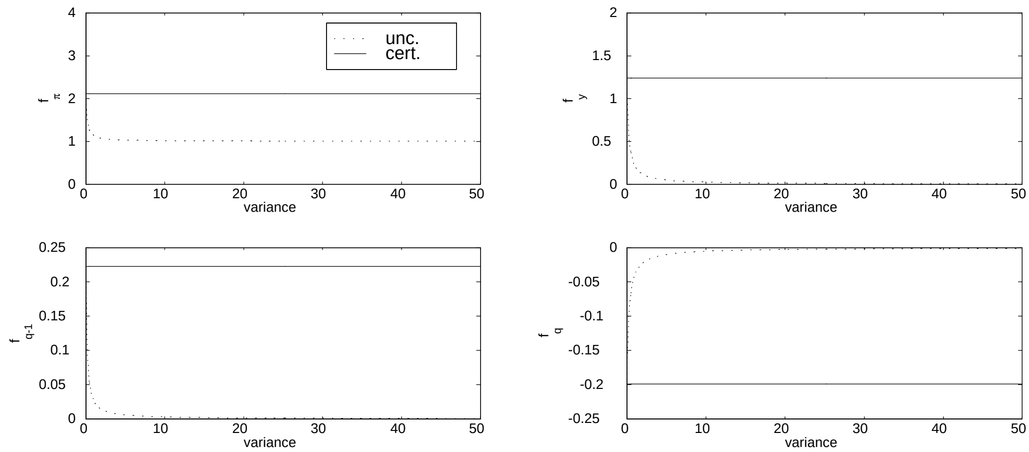

Figure 2: Effects of increasing β_q uncertainty. The top-left panel is the optimal response to inflation as uncertainty increases. The top-right panel depicts the optimal response of output; the bottom-left panel shows the response of the lagged real exchange rate; and, finally, the bottom-right panel depicts the optimal response of the current real exchange rate.

go from negative to positive values.[9]

In Figure 2, the responses to uncertainty concerning the effect of the real exchange rate on aggregate demand are shown (β_q). In general, the response coefficients in Figure 2 are somewhat less sensitive to uncertainty compared to Figure 1. That is, the response coefficients tend to level off more slowly compared to uncertainty in α_q. The response coefficient to f_q^{PU} in the bottom-right panel approaches zero as the uncertainty increases, which is in contrast to the result in Figure 1. Otherwise, the response coefficients in Figure 2 are similar in shape to Figure 1.

Figure 3, displays the responses to increasing uncertainty in γ_r. In contrast to Figures 1 and 2, the optimal responses decrease very slowly with increased uncertainty. One way of explaining this phenomenon would be to focus on the fact that there are two channels through which monetary policy can affect inflation and output gap. If one channel is uncertain, in this case the coefficient in the interest parity relation, it may not have such a big effect since all the other channels are still assumed to be known with certainty. Therefore, there is little effect on optimal policy of introducing uncertainty in γ_r, something which corresponds well with the

[9]In the context of the results in Section 4, the fact that the uncertainty coefficient for inflation, f_π, "asymptotically" goes to one, instead of zero, has an effect on the percentage change between certainty equivalence and parameter uncertainty discussed above. Therefore, the percentage change for f_π^{PU} is likely to be understated. Similarly, the fact that f_q "asymptotically" approaches 0.5 as uncertainty increases serves to overstate the percentage change resulting from parameter uncertainty.

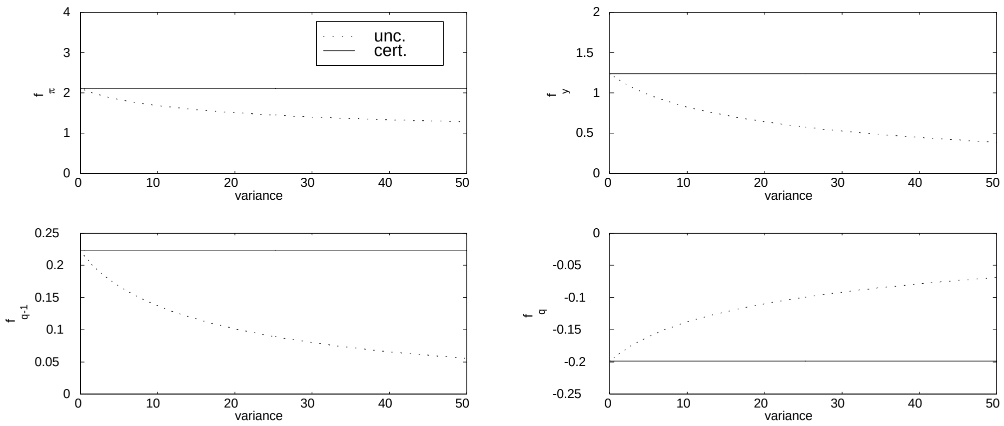

Figure 3: Effects of increasing γ_r uncertainty. The top-left panel is the optimal response to inflation as uncertainty increases. The top-right panel depicts the optimal response of output; the bottom-left panel shows the response of the lagged real exchange rate; and, finally, the bottom-right panel depicts the optimal response of the current real exchange rate.

results presented in Section 4.

Figures 1-3 present several interesting points. Increasing the uncertainty about the exchange rate pass-through to inflation causes the optimal response coefficients to decrease rapidly (and in one case, f_q, to switch signs). A similar picture is presented for uncertainty about the real exchange rate's effect on aggregate demand. Introducing uncertainty causes rapid attenuation in the response. While uncertainty in α_q and β_q are shown to have a relatively large effect on the optimal response coefficients, uncertainty in γ_r of the interest parity relation has little effect on the optimal response coefficients under uncertainty.

5 Empirical Results

To put the results of the preceeding sections in an empirical context, I estimate the model (7)-(9) on annual data from Germany, 1975-1998.[10] Over this period, the Bundesbank established an outstanding record in fighting inflation and safeguarding the value of the Deutsche Mark. While the Bundesbank nominally has been a monetary targeter, meaning that the money supply is the intermediary target, many researchers argue that Bundesbank has in fact engaged in inflation targeting (see, for example, Clarida and Gertler (1996), Bernanke and Mihov (1997)

[10]Quarterly data are readily available, but annual data are selected to correspond more closely to the theoretical model.

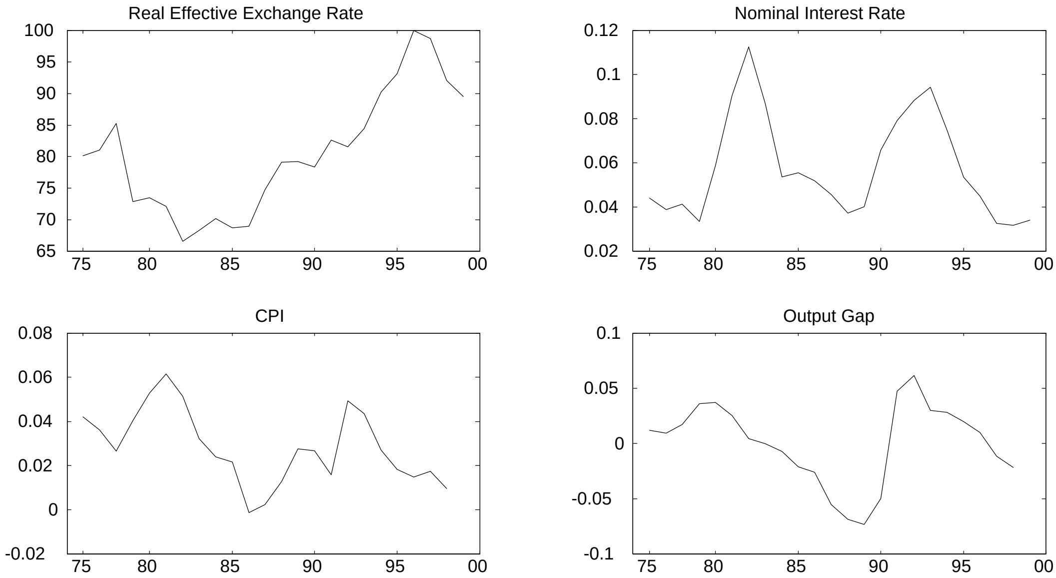

Figure 4: Data used in the empirical analysis

and Svensson (1999)).[11]

The purpose of this Section is only to give an empirical flavor of the theoretical results in the preceeding sections. Therefore, the empirical results should be viewed as illustrative "back-of-the-envelope" calculations, where many econometeric problems are glossed over. Nevertheless, the exercise can be viewed as an indication of how the results of the previous section compares to an actual estimated model.

The data are from the International Monetary Fund's International Financial Statistics (December, (1999)), and presented in Figure 4. The real effective exchange rate is based on relative normalized unit labor costs. The interest rate used is the annual average of the money market rate (MMR). The real output gap is measured as the log deviation from an Hodrick-Prescott trend.[12] Inflation is the annual percentage change in the consumer price index.

The model in (7)-(9) is estimated by OLS on an equation by equation basis. The results are presented in Table 2.

[11] While Germany certainly is an open economy, one may argue that it is not the stereotypically small economy often alluded to in theoretical analysis. However, due to the continuity of Bundesbank's monetary framework with emphasis on a floating exchange rate and stable monetary policy, Germany serves as a good starting point for the issues considered in this paper.

[12] The HP trend of the real output gap is calculated with smoothing factor of 100, appropriate for annual data.

Table 2: OLS Estimates of Baseline Model for Germany, 1975-1998

Parameter	Coefficient	Standard Error	R^2
$\alpha_{t+1,\pi}$	0.499	0.18	
$\alpha_{t+1,y}$	0.1234	0.08	
$\alpha_{t+1,q}$	0.079	0.046	$R^2 = 0.5882$
$\beta_{t+1,y}$	0.75	0.149	
$\beta_{t+1,r}$	0.24	0.323	
$\beta_{t+1,q}$	-0.02	0.046	$R^2 = 0.5858$
$\gamma_{t+1,r}$	-0.187	1.56	$R^2 = 0.0006$

The estimated coefficients have the theoretically expected signs, except for the coefficient on the real exchange rate in the aggregate demand equation, $\beta_{t+1,q}$, and the coefficient on the real interest rate in the interest parity relation, $\gamma_{t+1,r}$. Computing the t-statistics of the estimated empirical model and comparing to the t-values assumed in Section 4, the estimated t-values of the Phillips curve ($t^{est}_{\alpha_\pi} = 2.77$ vs. $t_{\alpha_\pi} = 2$, $t^{est}_{\alpha_y} = 1.53$ vs. $t_{\alpha_y} = 2$, and $t^{est}_{\alpha_q} = 1.71$ vs. $t_{\alpha_q} = 2$) are fairly close to the assumed values. However, the same cannot be said for the t-values of the estimated coefficients in the aggregate demand and interest parity relations ($t^{est}_{\beta_y} = 5.03$ vs. $t_{\beta_y} = 2$, $t^{est}_{\beta_r} = 0.74$ vs. $t_{\beta_r} = 2$, $t^{est}_{\beta_q} = 0.43$ vs. $t_{\beta_q} = 2$, and $t^{est}_{\gamma_r} = 0.11$ vs. $t_{\gamma_r} = 2$). In particular, the point estimates of β_r, β_q, and γ_r are far from the theoretically assumed values and estimated with large standard errors. In contrast, the lag on inflation, $\alpha_{t+1,\pi}$, is highly significant, as is $\beta_{t+1,y}$, the lag of output in the aggregate demand relation. Compared to the calibrated values used in Section 4, the point estimates in the empirical analysis are generally smaller. Furthermore, the explanatory power (R^2) in the interest parity relation (9) is very low, which corresponds well with the finding in Bagliano *et al.* (2001) that monetary policy has very little effect on exchange rates. The explanatory power for inflation in equation (7) and the output gap in equation (8) are much higher ($R^2 = 0.58$).

In the analysis below, the estimated coefficients and implied variances are used to conduct the same calculations as in Section 4. Thus, the only difference compared to Section 4 is that, instead of the values used by Ball (1999) and presented in Table 1, the estimated values for Germany in Table 2 are utilized. The standard deviation presented in Table 2 is squared to calculate the variance corresponding to uncertainty scenario number 4 in Table 1 (i.e. when all parameters are uncertain). As in Section 4, two cases will be considered: (1) the central bank calculates optimal policy without regard to the associated uncertainty (certainty equivalence) and then only uses the coefficient estimates in Table 2; and (2) the central bank computes optimal policy when the uncertainty in all parameters is taken into account in the minimization problem.

5.1 Reaction Function under Certainty

In this section, the point estimates presented in Table 2 are used to calculate the optimal reaction coefficients for Germany, $\{f_{\pi}^{CE}, f_{y}^{CE}, f_{q-1}^{CE}, f_{q}^{CE}\}$, which will be of the same form as equation (1). As in Section 4, λ is assumed to be equal to 0.5.

When the Bundesbank is not taking uncertainty into account, then the following optimal reaction function results:

$$i_t = 1.25\pi_t + 3.18y_t + 0.04q_{t-1} + 0.12q_t. \tag{18}$$

While the response coefficient on the inflation term, f_{π}^{CE}, is not too far from the value suggested by Taylor ($f_{\pi} = 1.5$), the response coefficient on the output gap is enormous ($f_{y}^{CE} = 3.18$), at least compared to what Taylor suggested ($f_y = 0.5$). The coefficients on the exchange rate variables are $f_{q-1}^{CE} = 0.04$ and $f_{q}^{CE} = 0.12$.

The optimal rule calculated in (18) can be used to find the Bundesbank implied reaction function for the period 1975-1998. By using the actual data for $\{\pi_t, y_t, q_{t-1}, q_t\}$ for the time period considered, an optimal path for the interest rate is constructed.[13]

The result of this exercise, when the central bank is not taking uncertainty into account, is presented in the top-left panel of Figure 5, where also the actual nominal interest rate (MMR) for the period is plotted for comparison.

From the top-left panel of Figure 5, it is clear that the estimated reaction function under certainty equivalence is far more aggressive than the actual interest setting by the Bundesbank, especially around the reunification. Note also that the scale is different on the top left panel to accomodate the extreme movements in the optimal path under certainty equivalence. Several other observers have noted that optimal certainty equivalent policy tends to be more aggressive than what is observed in practice, see for example Woodford (1999) and Söderström (1999). It is also interesting to note that what appears to be driving the results is the huge reaction coefficient on the output gap. This large reaction coefficient to the output gap is somewhat

[13] Söderström (1999) conducts a similar exercise for the closed-economy case of United States.

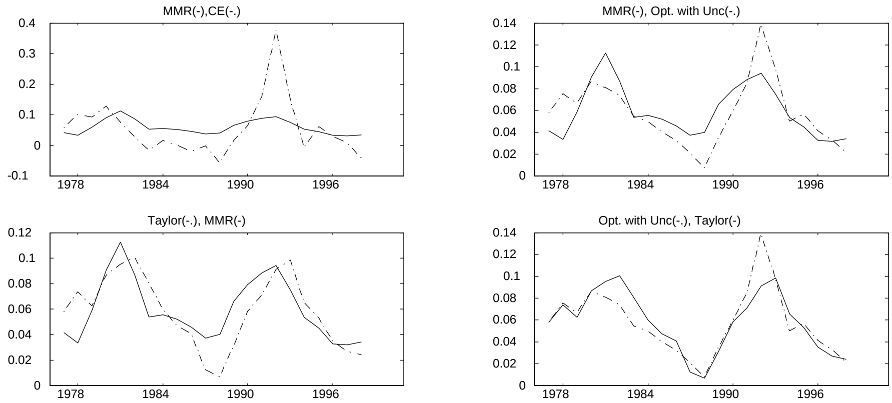

Figure 5: The top-left panel compares the Money Market Rate (MMR - solid line), determined by the Bundesbank, 1975-1998, to the optimal path for the interest rate under constant parameters (certainty equivalence, dash-dotted line); the top-right panel compares MMR (solid line) to the optimal path with uncertain parameters; the bottom-left panel describes the Taylor rule (dash-dotted line) and compares it to the actual MMR (solid line); finally, the bottom-right panel focuses on the Taylor rule (solid line), compared to the optimal path for the interest rate under uncertainty (dashed-dotted line).

surprising given that the weight on the output gap is only 0.5, i.e. the inflation objective is twice as important as the output gap objective.

5.2 Optimal Estimated Responses with Parameter Uncertainty

The optimal reaction function when uncertainty is accounted for in the optimization problem is

$$i_t = 1.06\pi_t + 0.9y_t + 0.01q_{t-1} + 0.03q_t. \tag{19}$$

The coefficients on the exchange rate variable are very small ($f_{q-1}^{PU} = 0.01$ and $f_q^{PU} = 0.03$); the optimal reaction coefficient on inflation, $f_\pi^{PU} = 1.06$, is smaller than suggested by Taylor, but slightly greater than one; the coefficient on the output gap, $f_y^{PU} = 0.9$, is higher than the value recommended by Taylor. The optimal reaction function is thus well approximated using only inflation and output gap – the real exchange rate plays a very small role and appears to be quantitatively insignificant.

The optimal reaction function in (19) can be utilized to calculate the optimal implied path for the German nominal interest rate (MMR) as in the preceeding subsection. The resulting path is presented in the top-right panel of Figure 5, where the reaction function under uncertainty (dashed-dotted line) is plotted against the actual interest rate set by the Bundesbank (MMR, solid line). Compared to the optimal path implied by certainty equivalence in the top-left panel of Figure 5, the implied optimal path under uncertainty in the top-right panel of Figure 5 does a good job of explaining the actual setting of the nominal interest rate by the Bundesbank.[14]

5.3 Comparing Optimal Policy under Uncertainty to the Taylor Rule

It is interesting to compare the actual interest rate set by the Bundesbank to what the Taylor rule would have implied for the period considered. With the optimal response coefficients on the exchange rate being close to zero, when uncertainty is considered, there is reason to believe that Taylor's rule would perform quite well.

The results of comparing the Taylor rule (dashed-dotted line) with response coefficients of $f_\pi = 1.5$ and $f_y = 0.5$, to actual German short-term nominal interest rate (MMR, solid line), are displayed in the bottom-left panel of Figure 5. Except for a period in the late 1980s and early 1990s, the Taylor rule provides a stellar fit for the period considered.

Finally, the Taylor rule can be compared to the optimal rule under uncertainty (see the bottom-right panel of Figure 5). The similarity is striking! The Taylor rule and the optimal rule under uncertainty are quite similar, at least based on visual inspection. The optimal rule under uncertainty is a little more aggressive in the wake of the reunification, which may be explained by the relatively large coefficient on the output gap in (19).

To summarize the empirical section, the optimal rule under uncertainty explains the actual behavior by the Bundesbank quite well, at least compared to the optimal rule calculated under certainty equivalence. Furthermore, the optimal rule under uncertainty contains a very small role for the exchange rate variables, thus rendering the policy rule close to the Taylor rule. Of course, these results must be interpreted with at least a modicum of caution. The estimation technique is crude and the observations are few. Perhaps more sophisticated estimation methods can unearth an increased role for the exchange rate in an estimated optimal reaction function.

[14] The effects of the uncertainty may be overestimated due to the neglect of covariances.

6 Summary and Discussion

Monetary policy in open economies is currently an active field of research. As discussed above, there is no consensus of how and if a central bank should respond to exchange rate variables in the monetary transmission mechanism. While many proposals of open-economy rules exist, there are relatively few discussions of the associated uncertainty about exchange rate variables.

The main result of this paper is that uncertainty is important in any discussion of monetary policy rules in open economies. When faced with exchange rate uncertainty, the policy makers at the central bank become more cautious in line with the “Brainard conservatism principle.” It is shown in the paper that uncertainty can cause a dramatic reduction in the optimal response coefficients on the exchange rate variables, compared to the benchmark case of no uncertainty. Thus, when realistic uncertainty about the exchange rate variables is taken into account, it may well be that it is optimal for the central bank to set the interest rate rule only as a function of the output gap and the inflation, as prescribed by the Taylor rule. This conclusion agrees with the results that Leitemo and Söderström (2001) reached in a different model set-up and different assumptions about the uncertainty facing the central bank.

Should monetary policy ignore exchange rates? Despite posing the question in the title, I will not take a definite stance on the question. Rather, the answer offered here is that monetary policy should take uncertainty into account. Given that uncertainty about exchange rate variables in the monetary transmission mechanism makes policy makers more cautious, at least in this problem formulation, it is conceivable that a Taylor rule is a good approximation to optimal monetary policy even in an open economy.

The absence of a definite conclusion is due to the many caveats that apply to the analysis in this paper. It is of course possible to construct an extremely open economy where the real exchange rate is likely to play an important role, even when realistic parameter uncertainty is taken into account. That is, the results presented above are to a certain extent dependent on the parameterization of the model.

The results are also driven by the nature of uncertainty considered in this paper. In the robust-control strand of research on monetary policy under model uncertainty, a standard result is that it is often optimal to react *more* strongly to uncertainty compared to certainty equivalence, which is in stark contrast to the standard results coming out of the Bayesian parameter uncertainty approach (considered in this paper). If one were to consider a similar problem in a robust-control framework, the optimal response to uncertainty about the exchange rate parameters would likely be *stronger* than certainty equivalence, and the line of reasoning used in this paper would have to be changed. However, the intutitive appeal of the robust-control literature is not as strong as the Brainard conservatism principle, as argued by Blinder (1998).

The absence of microfoundations clearly limits the usefulness of the policy advice coming out of this paper. That is, the modeling strategy as well as the empirical evaluation performed here, are subject to the Lucas Critique. With adequate microfoundations many more interesting questions could be analyzed such as the welfare properties of alternative rules under parameter uncertainty. The main reason why a forward-looking, microfounded model is not used here is the difficulty of introducing parameter uncertainty in such a framework.

Yet, regardless of the parameterization, the absence of microfoundations and the modeling of uncertainty, the conclusion that any discussion of monetary policy in an open economy will need to consider the associated uncertainty is likely be robust.

A Appendix

This appendix contains a detailed solution to the optimal control problem under certainty equivalence and parameter uncertainty.

A.1 Constant Parameters (Certainty Equivalence)

Solving the stochastic minimization problem with constant parameters involves choosing an interest rate, i_t, to minimize the loss function of the central bank

$$\min_{i_t} \sum_{t=0}^{\infty} \beta^t \left\{ x_t' Q x_t \right\}, \qquad 0 < \beta < 1$$

subject to the law of motion

$$x_{t+1} = Ax_t + Bi_t + \varepsilon_{t+1}, \;\; t \geq 0$$

where ε_{t+1} is an $n \times 1$ vector of random variables drawn from a normal distribution with mean zero and covariance matrix

$$E\varepsilon_t \varepsilon_t' = \Sigma.$$

The value function for a model with quadratic goal function and linear constraints is

$$v(x_t) = x_t' V x_t + d, \tag{A1}$$

where V is the unique negative semidefinite solution to a Riccati equation and $d = \frac{\beta}{1-\beta} trV\Sigma$, which will be further discussed below. The Bellman equation is given by

$$v(x_t) = \min_{i_t} \left[x_t' Q x_t + \beta E_t v(x_{t+1}) \right]. \tag{A2}$$

The optimal solution can be found by substituting our guess of the value function, (A1), into the Bellman equation (A2)

$$v(x_t) = \min_{i_t} \left\{ x_t' Q x_t + \beta E_t \left[(Ax_t + Bi_t + \varepsilon_{t+1})' V (Ax_t + Bi_t + \varepsilon_{t+1}) \right] + d \right\}. \tag{A3}$$

Expanding the quadratic form of (A3) and utilizing the properties of transposed matrices, $(AB)' = B'A'$, give

$$\begin{aligned} v(x_t) \;\; = \;\; & \min_{i_t} \{ x_t' Q x_t + \beta E_t \{ x_t' A' V A x_t + x_t' A' V B i_t + x_t' A' V \varepsilon_{t+1} \\ & + i_t' B' V A x_t + i_t' B' V' B i_t + i_t' B' V \varepsilon_{t+1} \\ & + \varepsilon_{t+1}' V A x_t + \varepsilon_{t+1}' V B i_t + \varepsilon_{t+1}' V \varepsilon_{t+1} + d \} \end{aligned} \tag{A4}$$

Applying the expectations operator yields

$$\begin{aligned} v(x_t) &= \min_{i_t}\{x_t'Qx_t + \beta x_t'A'VAx_t + \beta x_t'A'VBi_t + \\ & \beta i_t'B'VAx_t \\ & \beta i_t'B'VBi_t + \beta E_t\varepsilon_{t+1}'V\varepsilon_{t+1}\} + \beta d. \end{aligned} \tag{A5}$$

In taking the first order conditions with respect to i_t, the following matrix derivation rules from Ljungqvist and Sargent (2000) are used:

$$\frac{\partial x'Ax}{\partial x} = (A+A')x, \tag{A6}$$

$$\frac{\partial y'Az}{\partial y} = Bz, \tag{A7}$$

$$\frac{\partial y'Bz}{\partial z} = B'y. \tag{A8}$$

The first-order condition with respect to i_t is then

$$\begin{aligned} \frac{\partial v(x_t)}{\partial i_t} &= \beta((A'VB)'x_t + \beta(B'VA)x_t + \beta((B'VB + (B'VB)')i_t = 0 \\ 0 &= B'\left(V+V'\right)Ax_t + B'(V+V')Bi_t. \end{aligned}$$

Solving for the optimal setting of the instrument, i_t, yields

$$\begin{aligned} i_t &= -\left(B'\left(V+V'\right)B\right)^{-1}B'\left(V+V'\right)Ax_t, \\ i_t &= F^{CE}x_t \\ i_t &= f_\pi^{CE}\pi_t + f_y^{CE}y_t + f_{q-1}^{CE}q_{t-1} + f_q^{CE}q_t. \end{aligned} \tag{A9}$$

which corresponds to equation (16) in Section 3. Equation (A9) shows that optimal policy is independent of any uncertainty, thus coinciding with the deterministic solution to the linear-quadratic problem.

Substituting the policy rule, (A9), into the right-hand side of equation (A3) gives

$$\begin{aligned} x_tVx_t + d &= \left\{x_t'Qx_t + \beta E_t\left[(Ax_t + Bfx_t + \varepsilon_{t+1})'V(Ax_t + Bfx_t + \varepsilon_{t+1})\right] + d\right\} \\ &= \{x_t'Qx_t + \beta E_t(x_t'A'VAx_t + x_t'A'VBfx_t + x_t'A'V\varepsilon_{t+1} + \\ & \quad x_t'f'B'VAx_t + x_t'f'B'VBx_t + x_t'f'B'\varepsilon_{t+1} \\ & \quad \varepsilon_{t+1}'VAx_t + \varepsilon_{t+1}'VBfx_t + \varepsilon_{t+1}'V\varepsilon_{t+1}) + d\}. \end{aligned}$$

Applying the expectations operator yields

$$x_tVx_t + d = \left\{x_t'Qx_t + \beta(Ax_t + Bfx_t)'V(Ax_t + Bfx_t) + trV\Sigma + d\right\}, \tag{A10}$$

where the formula $E\varepsilon'V\varepsilon = trE\varepsilon'V\varepsilon = trVE\varepsilon\varepsilon' = trV\Sigma$ is used and tr denotes the trace operator. Rearranging (A10) produces

$$x_tVx_t + d = x_t'[Q + \beta(A+Bf)'V(A+Bf)]x_t + \beta\left(trV\Sigma + d\right),$$

where $d = \frac{\beta}{1-\beta} trV\Sigma$, and the V matrix is determined by the algebraic matrix Riccati equation

$$V = Q + \beta(A + Bf)'V(A + Bf). \tag{A11}$$

Whenever V is larger than a 2×2 matrix, numerical solutions must be used. See Ljungqvist and Sargent (2000, ch. 4) for more details and proof of certainty equivalence.

A.2 Parameter Uncertainty

With uncertain parameters, variances of the parameters in the problem need to be considered. Recall that the stochastic parameter matrices can be expressed as

$$\begin{aligned} A_{t+1} &= A + \varepsilon^A_{t+1}, && \text{(A12)} \\ B_{t+1} &= B + \varepsilon^B_{t+1}, && \text{(A13)} \end{aligned}$$

where A and B are the mean of the matrices in (12); ε^A_{t+1} is a 4×4 matrix of coefficient shock terms with mean zero, and covariance matrix, $E_t \varepsilon^A_{t+1} \varepsilon^{A\prime}_{t+1} = \Sigma^A$; ε^B is 4×1 vector of coefficient shock terms with mean zero, and covariance matrix $E_t \varepsilon^B_{t+1} \varepsilon^{B\prime}_{t+1} = \Sigma^B$. All the coefficient shock terms are assumed to be independent of each other and the structural shocks.

More concretely, the matrices (A12) and (A13) can be expressed as

$$\begin{aligned}
A_{t+1} &= \begin{bmatrix} \alpha_\pi + \varepsilon^{\alpha_\pi} & \alpha_y + \varepsilon^{\alpha_y} & \alpha_q + \varepsilon^{\alpha_q} & -\left(\alpha_q + \varepsilon^{\alpha_q}\right) \\ \beta_r + \varepsilon^{\beta_r} & \beta_y + \varepsilon^{\beta_y} & 0 & -\left(\beta_q + \varepsilon^{\beta_q}\right) \\ 0 & 0 & 0 & 1 \\ -\left(\gamma_r + \varepsilon^{\gamma_r}\right) & 0 & 0 & 0 \end{bmatrix}, \\
E_t A_{t+1} &= \begin{bmatrix} \alpha_\pi & \alpha_y & \alpha_q & -\alpha_q \\ \beta_r & \beta_y & 0 & -\beta_q \\ 0 & 0 & 0 & 1 \\ -\gamma_r & 0 & 0 & 0 \end{bmatrix}, \ \varepsilon^A_{t+1} = \begin{bmatrix} \varepsilon^{\alpha_\pi} & \varepsilon^{\alpha_y} & \varepsilon^{\alpha_q} & -\varepsilon^{\alpha_q} \\ \varepsilon^{\beta_r} & \varepsilon^{\beta_y} & 0 & -\varepsilon^{\beta_q} \\ 0 & 0 & 0 & 0 \\ -\varepsilon^{\gamma_r} & 0 & 0 & 0 \end{bmatrix}, \\
B_{t+1} &= \begin{bmatrix} 0 \\ -\left(\beta_r + \varepsilon^{\beta_r}\right) \\ 0 \\ \left(\gamma_r + \varepsilon^{\gamma_r}\right) \end{bmatrix}, \ E_t B_{t+1} = \begin{bmatrix} 0 \\ -\beta_r \\ 0 \\ \gamma_r \end{bmatrix}, \ \varepsilon^B_{t+1} = \begin{bmatrix} 0 \\ -\varepsilon^{\beta_r} \\ 0 \\ \varepsilon^{\gamma_r} \end{bmatrix}.
\end{aligned}$$

Following a similar strategy as outlined in A.1, equation (A3) is now written with stochastic matrices. Again, expanding the quadratic form gives

$$\begin{aligned} v(x_t) = {} & \max_{i_t} \{ x_t' Q x_t + \beta E_t \{ x_t' A_{t+1}' V A_{t+1} x_t + x_t' A_{t+1}' V B_{t+1} i_t + x_t' A_{t+1}' V \varepsilon_{t+1} \\ & + i_t' B_{t+1}' V A_{t+1} x_t + i_t' B_{t+1}' V' B_{t+1} i_t + i_t' B_{t+1}' V \varepsilon_{t+1} \\ & + \varepsilon_{t+1}' V A_{t+1} x_t + \varepsilon_{t+1}' V B_{t+1} i_t + \varepsilon_{t+1}' V \varepsilon_{t+1} + d \}. \end{aligned} \tag{A15}$$

Using the expressions in (A12) and (A13), the preceeding equation (A15) is written as

$$\begin{aligned}
v(x_t) &= \min_{i_t}\{x_t'Qx_t + \beta E_t\{x_t'\left(A+\varepsilon_{t+1}^A\right)'V\left(A+\varepsilon_{t+1}^A\right)x_t + \\
&\quad x_t'\left(A+\varepsilon_{t+1}^A\right)'V\left(B+\varepsilon_{t+1}^B\right)i_t + x_t'\left(A+\varepsilon_{t+1}^A\right)'V\varepsilon_{t+1} \\
&\quad +i_t'\left(B+\varepsilon_{t+1}^B\right)'V\left(A+\varepsilon_{t+1}^A\right)x_t + i_t'\left(B+\varepsilon_{t+1}^B\right)V'\left(B+\varepsilon_{t+1}^B\right)i_t \\
&\quad +i_t'\left(B+\varepsilon_{t+1}^B\right)'V\varepsilon_{t+1} + \varepsilon_{t+1}'V\left(A+\varepsilon_{t+1}^A\right)x_t + \varepsilon_{t+1}'V\left(B+\varepsilon_{t+1}^B\right)i_t \\
&\quad +\varepsilon_{t+1}'V\varepsilon_{t+1} + d\}.
\end{aligned} \tag{A16}$$

Expanding and applying the expectations operator (A16) becomes

$$\begin{aligned}
v(x_t) &= \min_{i_t}\{x_t'Qx_t + \beta\{E_tx_t'A'VAx_t + E_tx_t'A'VBi_t \\
&\quad +E_ti_t'B'VAx_t + E_ti_t'B'VBi_t \\
&\quad +E_tx_t'\varepsilon_{t+1}^{A\prime}V\varepsilon_{t+1}^Ax_t && \text{(A17)} \\
&\quad +E_ti_t'\varepsilon_{t+1}^{B\prime}V\varepsilon_{t+1}^Ax_t && \text{(A18)} \\
&\quad +E_tx_t'\varepsilon_{t+1}^{A\prime}V\varepsilon_{t+1}^Bi_t && \text{(A19)} \\
&\quad +E_ti_t'\varepsilon_{t+1}^{B\prime}V\varepsilon_{t+1}^Bi_t && \text{(A20)} \\
&\quad +E_t\varepsilon_{t+1}'V\varepsilon_{t+1} + d\}. && \text{(A21)}
\end{aligned}$$

Compared to the case of constant parameters (A5), four extra terms appears due to parameter uncertainty, as outlined in (A17) to (A20). Below each term of (A17) to (A20) is analyzed individually.

Before plunging into the analysis of each term, the following matrix formula is used repeatedly (see Ljungqvist and Sargent (2000):

$$\begin{aligned}
E_t\varepsilon_{t+1}'V\varepsilon_{t+1} &= trE_t\varepsilon_{t+1}'V\varepsilon_{t+1}, \\
&= trVE_t\varepsilon_{t+1}\varepsilon_{t+1}', \\
&= trV\Sigma.
\end{aligned} \tag{A22}$$

Term A17 Starting with term (A17) and applying the formula in (A22) produces

$$\begin{aligned}
E_tx_t'\left(\varepsilon_{t+1}^A\right)'V\left(\varepsilon_{t+1}^A\right)x_t &= trE_tx_t'\left(\varepsilon_{t+1}^A\right)'V\left(\varepsilon_{t+1}^A\right)x_t, \\
&= trVE_t\left(\varepsilon_{t+1}^A\right)x_tx_t'\left(\varepsilon_{t+1}^A\right)', \\
&= v_{11}\left(\varepsilon^{\alpha_\pi}\pi_t + \varepsilon^{\alpha_y}y_t + \varepsilon^{\alpha_q}q_{t-1} - \varepsilon^{\alpha_q}q_t\right)^2 \\
&\quad +v_{22}\left(\varepsilon^{\beta_r}\pi_t + \varepsilon^{\beta_y}y_t - \varepsilon^{\beta_q}q_t\right)^2 + v_{44}\left(\varepsilon^{\gamma_r}\right)^2\pi_t^2.
\end{aligned}$$

Term A18 The second term, (A18), using (A22), becomes

$$\begin{aligned}
E\left(i_t'(\varepsilon_{t+1}^B)'V\left(\varepsilon_{t+1}^A\right)x_t\right) &= trE\left(i_t'(\varepsilon_{t+1}^B)'V\left(\varepsilon_{t+1}^A\right)x_t\right), \\
&= trVE\left(\varepsilon_{t+1}^A\right)x_tVi_t'(\varepsilon_{t+1}^B)', \\
&= -v_{22}\pi_t\left(\varepsilon^{\beta_r}\right)^2i_t - v_{44}\pi_t\left(\varepsilon^{\gamma_r}\right)^2i_t.
\end{aligned}$$

Term A19 The third term is (A19) which, combined with (A22), can be expressed as

$$\begin{aligned} E_t x_t' \left(\varepsilon_{t+1}^A\right)' V(\varepsilon_{t+1}^B) i_t &= trE_t x_t' \left(\varepsilon_{t+1}^A\right)' V(\varepsilon_{t+1}^B) i_t, \\ &= trVE_t(\varepsilon_{t+1}^B) i_t x_t' \left(\varepsilon_{t+1}^A\right)', \\ &= -v_{22}\pi_t \left(\varepsilon^{\beta_r}\right)^2 i_t - v_{44}\pi_t \left(\varepsilon^{\gamma_r}\right)^2 i_t. \end{aligned}$$

Term A20 Finally, term (A20), using (A22), can be written as

$$\begin{aligned} E_t \left(i_t'(\varepsilon^B)' V(\varepsilon^B) i_t\right) &= trE_t \left(i_t'(\varepsilon^B)' V(\varepsilon^B) i_t\right), \\ &= trVE_t(\varepsilon^B) i_t i_t'(\varepsilon^B)', \\ &= v_{22} i_t \left(\varepsilon^{\beta_r}\right)^2 i_t + v_{44} i_t \left(\varepsilon^{\gamma_r}\right)^2 i_t. \end{aligned}$$

Before summing (A17) to (A20), the error vector takes the following form:

$$\varepsilon_{t+1} = \begin{bmatrix} \epsilon_{t+1} \\ \eta_{t+1} \\ 0 \\ \nu_{t+1} \end{bmatrix}.$$

Using our trace formula (A22) on (A21) produces

$$\begin{aligned} E\varepsilon_{t+1}' V \varepsilon_{t+1} &= trE\varepsilon_{t+1}' V \varepsilon_{t+1} \\ &= trVE\varepsilon_{t+1}\varepsilon_{t+1}' \\ &= trV\Sigma, \\ &= v_{11}\epsilon_{t+1}^2 + v_{22}\eta_{t+1}^2 + v_{44}\nu_{t+1}^2. \end{aligned}$$

Adding up the Terms (A17-A21) Adding up the terms which arise due to parameter uncertainty in (A17) to (A20) and the structural shocks in (A21), then yields the sum

$$\begin{aligned} & v_{11}\left(\varepsilon^{\alpha_\pi}\pi_t + \varepsilon^{\alpha_y} y_t + \varepsilon^{\alpha_q} q_{t-1} - \varepsilon^{\alpha_q} q_t\right)^2 + v_{22}\left(\varepsilon^{\beta_r}\pi_t + \varepsilon^{\beta_y} y_t - \varepsilon^{\beta_q} q_t\right)^2 \\ & + v_{44}\left(\varepsilon^{\gamma_r}\right)^2 \pi_t^2 \\ & - v_{22}\pi_t \left(\varepsilon^{\beta_r}\right)^2 i_t - v_{44}\pi_t \left(\varepsilon^{\gamma_r}\right)^2 i_t \\ & - v_{22}\pi_t \left(\varepsilon^{\beta_r}\right)^2 i_t - v_{44}\pi_t \left(\varepsilon^{\gamma_r}\right)^2 i_t + \\ & v_{22} i_t \left(\varepsilon^{\beta_r}\right)^2 i_t + v_{44} i_t \left(\varepsilon^{\gamma_r}\right)^2 i_t + \\ & v_{11}\epsilon_{t+1}^2 + v_{22}\eta_{t+1}^2 + v_{44}\nu_{t+1}^2. \end{aligned}$$

Collecting terms and writing out the variances,

$$\begin{aligned} & v_{11}\left[\pi_t\sigma_{\alpha_\pi}^2\pi_t + y_t\sigma_{\alpha_y}^2 y_t + q_{t-1}\sigma_{\alpha_q}^2 q_{t-1} + q_t\sigma_{\alpha_q}^2 q_t - q_{t-1}\sigma_{\alpha_q}^2 q_t - q_t\sigma_{\alpha_q}^2 q_{t-1} + \sigma_\epsilon^2\right] \\ + \quad & v_{22}\left[\pi_t\sigma_{\beta_r}^2\pi_t + y_t\sigma_{\beta_y}^2 y_t + q_t\sigma_{\beta_q}^2 q_t - 2\pi_t\sigma_{\beta_r}^2 i + i\sigma_{\beta_r}^2 i + \sigma_\eta^2\right] \\ + \quad & v_{44}\left[\pi_t\sigma_{\gamma_r}^2\pi_t - 2\pi_t\sigma_{\gamma_r}^2 i + i\sigma_{\gamma_r}^2 i + \sigma_\nu^2\right]. \end{aligned}$$

Collecting the variances in matrices

$$
\begin{aligned}
\Sigma_{11}^{A} &= \begin{bmatrix} \sigma_{\alpha_\pi}^2 & 0 & 0 & 0 \\ 0 & \sigma_{\alpha_y}^2 & 0 & 0 \\ 0 & 0 & \sigma_{\alpha_q}^2 & -\sigma_{\alpha_q}^2 \\ 0 & 0 & -\sigma_{\alpha_q}^2 & \sigma_{\alpha_q}^2 \end{bmatrix}, \Sigma_{22}^{A} = \begin{bmatrix} \sigma_{\beta_r}^2 & 0 & 0 & 0 \\ 0 & \sigma_{\beta_y}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{\beta_q}^2 \end{bmatrix}, \\
\Sigma_{22}^{AB} &= \begin{bmatrix} \sigma_{\beta_r}^2 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \Sigma_{44}^{AB} = \begin{bmatrix} \sigma_{\gamma_r}^2 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \Sigma_{44}^{A} = \begin{bmatrix} \sigma_{\gamma_r}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \\
\Sigma_{22}^{B} &= \sigma_{\beta_r}^2, \Sigma_{44}^{B} = \sigma_{\gamma_r}^2,
\end{aligned}
$$

the extra term due to parameter uncertainty is compactly expressed as

$$
\begin{aligned}
& v_{11}\left[x_t'\Sigma_{11}^{A}x_t + \Sigma_\epsilon\right] + \\
& v_{22}\left[x_t'\Sigma_{22}^{A}x_t + 2x_t'\Sigma_{22}^{AB}i_t + i_t'\Sigma_{22}^{B}i_t + \Sigma_\eta\right] + \\
& v_{44}\left[x_t'\Sigma_{44}^{A}x_t + 2x_t'\Sigma_{44}^{AB}i_t + i_t'\Sigma_{44}^{B}i_t + \Sigma_\nu\right].
\end{aligned}
$$

Finally, the expected value of the Bellman equation with parameter uncertainty is then

$$
\begin{aligned}
v(x_t) &= \min_{i_t}[x_t'Qx_t + \beta E_t v(x_{t+1})] \\
&= \min_{i_t}[x_t'Qx_t + \beta\{(Ax_t + Bi_t)'V(Ax_t + Bi_t) + \\
&\quad v_{11}(x_t'\Sigma_{11}^{A}x_t + \Sigma_\epsilon) + \\
&\quad v_{22}(x_t'\Sigma_{22}^{A}x_t + 2x_t'\Sigma_{22}^{AB}i + i'\Sigma_{22}^{B}i + \Sigma_\eta) + \\
&\quad v_{44}(\pi_t'\Sigma_{44}^{A}\pi_t + 2x_t'\Sigma_{44}^{AB}i + i'\Sigma_{44}^{B}i + \Sigma_\nu) + d\}].
\end{aligned}
$$

Multiplying out the quadratic form gives

$$
\begin{aligned}
v(x_t) &= \min_{i_t}[x_t'Qx_t + \beta\{x_t'A'VAx_t + x_t'A'VBi_t \\
&\quad + i_t'B'VAx_t + i_t'B'VBi_t + \\
&\quad v_{11}(x_t'\Sigma_{11}^{A}x_t + \Sigma_\epsilon) + \\
&\quad v_{22}(x_t'\Sigma_{22}^{A}x_t + 2x_t'\Sigma_{22}^{AB}i + i'\Sigma_{22}^{B}i + \Sigma_\eta) + \\
&\quad v_{44}(x_t'\Sigma_{44}^{A}x_t + 2x_t'\Sigma_{44}^{AB}i + i'\Sigma_{44}^{B}i + \Sigma_\nu) + d\}].
\end{aligned}
$$

Using the matrix derivation rules in (A6) to (A8), the first order condition for i_t is

$$
\begin{aligned}
0 &= \beta(B'V'Ax_t + B'VAx_t + B'(V + V')Bi_t + \\
&\quad 2v_{22}\Sigma_{22}^{AB\prime}x_t + v_{22}(\Sigma_{22}^{B} + \Sigma_{22}^{B\prime})i_t + 2v_{44}\Sigma_{44}^{AB\prime}x_t + v_{44}(\Sigma_{44}^{B} + \Sigma_{44}^{B\prime})i_t).
\end{aligned}
$$

Solving for i_t yields the optimal interest rate path,

$$
\begin{aligned}
i_t &= -[B'(V+V')A + 2v_{22}\Sigma_{22}^{AB\prime} + 2v_{44}\Sigma_{44}^{AB\prime}]x_t \times \\
&\quad \left(B'(V+V')B + 2v_{22}\Sigma_{22}^{B} + 2v_{44}\Sigma_{44}^{B}\right)^{-1} \\
i_t &= F^{PU}x_t \\
i_t &= f_{\pi}^{PU}\pi_t + f_{y}^{PU}y_t + f_{q-1}^{PU}q_{t-1} + f_{q}^{PU}q_t, \qquad \text{(A23)}
\end{aligned}
$$

which corresponds to equation (17) in Section 3. Comparing (A23) to (A9), it is clear that certainty equivalence fails under parameter uncertainty; optimal policy in (A23) is dependent on the variance-covariance terms of the parameter matrices.

Using the optimal interest rate path (A23) in the Bellman equation produces

$$
\begin{aligned}
& x_t'Vx_t + d \\
&= \left[x_t'Qx_t + \beta \left\{ \begin{array}{c} (Ax_t + Bfx_t)'\,V\,(Ax_t + Bfx_t) + v_{11}(x_t'\Sigma_{11}^{A}x_t + \Sigma_{\epsilon}) + \\ v_{22}(x_t'\Sigma_{22}^{A}x_t + 2x_t'\Sigma_{22}^{AB}i + i'\Sigma_{22}^{B}i + \Sigma_{\eta}) + \\ v_{44}(x_t'\Sigma_{44}^{A}x_t + 2x_t'\Sigma_{44}^{AB}i + i'\Sigma_{44}^{B}i + \Sigma_{\nu}) + d \end{array} \right\} \right] \\
&= x_t' \left[Q + \beta \left\{ \begin{array}{c} (A+Bf)'\,V\,(A+Bf) + v_{11}\Sigma_{11}^{A} + \\ v_{22}(\Sigma_{22}^{A} + 2\Sigma_{22}^{AB}f + f\Sigma_{22}^{B}f) + \\ v_{44}(\Sigma_{44}^{A} + 2\Sigma_{44}^{AB}f + f\Sigma_{44}^{B}f) \end{array} \right\} \right] x_t + \\
& \beta\left(v_{11}\Sigma_{\epsilon} + v_{22}\Sigma_{\eta} + v_{44}\Sigma_{\nu} + d\right),
\end{aligned}
$$

which implies that the matrix V is determined by

$$
V = \left[Q + \beta \left\{ \begin{array}{c} (A+Bf)'\,V\,(A+Bf) + v_{11}\Sigma_{11}^{A} + \\ v_{22}(\Sigma_{22}^{A} + 2\Sigma_{22}^{AB}f + f\Sigma_{22}^{B}f) + \\ v_{44}(\Sigma_{44}^{A} + 2\Sigma_{44}^{AB}f + f\Sigma_{44}^{B}f) \end{array} \right\} \right].
$$

References

[1] Bagliano, F., C. Favero and F. Franco (2001), "Measuring Monetary Policy in Open Economies," Working Paper.

[2] Ball, L. (1999), "Policy Rules for Open Economies," in J. Taylor (ed.) *Monetary Policy Rules*, University of Chicago Press.

[3] Batini, N., R. Harrison and S. Millard (2001), "Monetary Policy Rules for an Open Economy," Paper presented at the San Francisco conference on monetary policy and exchange rates, April.

[4] Bernanke, B. and I. Mihov (1997), "What Does the Bundesbank Target?" *European Economic Review* 41, pp. 1025-1054.

[5] Blinder, A. (1998), *Central Banking in Theory and Practice*, MIT Press.

[6] Brainard, W. (1967), "Uncertainty and the Effectiveness of Policy," *American Economic Review 57, Papers and Proceedings*, pp. 411-425.

[7] Chinn, M. and E. Fujii (2001), "Fin de Siecle Real Interest Parity," NBER Working Paper No. 7880.

[8] Clarida, R, and M. Gertler (1996), "How the Bundesbank Conducts Monetary Policy," NBER Working paper No. 5581.

[9] Dennis, R. (2000), "Optimal Simple Rules for Small Open Economies," Federal Reserve Bank of San Francisco Working Paper 00/20.

[10] Eichenbaum and Evans (1995) "Some Empirical Evidence on the Effects of Shocks to Monetary Policy on Exchange Rates," *Quarterly Journal of Economics*, Vol. 110, Issue 4, pp. 975-1009.

[11] Estrella, A. and J. Fuhrer (1999) "Are Deep Parameters Stable? The Lucas Critique as an Empirical Hypothesis," Working Paper No. 99-7, Federal Reserve Bank of Boston. Forthcoming *American Economic Review.*

[12] Frankel, J. and K. Froot (1987), "Using Survey Data to Test Standard Propositions Regarding Exchange Rate Expectations," *American Economic Review*, Vol 77. March, pp. 133-153.

[13] Giannoni, M. (2000), "Does Model Uncertainty Justify Caution? Robust Optimal Monetary Policy in a Forward-Looking Model," Mimeo, Princeton University. Forthcoming in *Macroeconomic Dynamics.*

[14] Goldberg, P. and M. Knetter (1997), "Goods Prices and Exchange Rates: What Have We Learned?" *Journal of Economic Literature,* September, pp. 1243-1272.

[15] Hansen, L.P. and T. Sargent (2000), "Robust Control and Filtering of Forward-Looking models," Mimeo, Stanford University.

[16] Hodrick, R. (1987), *The Empirical Evidence on the Efficiency of Forward and Futures Exchange Markets*, Harcourt Academic Publishers.

[17] Leitemo, K. and U. Söderström (2001), "Simple Rules and Exchange Rate Uncertainty," Sveriges Riksbank Working Paper Series No. 123.

[18] Ljungqvist, L. and T. Sargent (2000), *Recursive Macroeconomic Theory*, MIT Press.

[19] Obstfeld, M. and K. Rogoff (1995), "The Mirage of Fixed Exchange Rates," *Journal of Economic Perspectives*, Fall, pp. 73-96.

[20] Sack, B. (2000), "Does the Fed Act Gradually? A VAR Analysis," *Journal of Monetary Economics* 46 (1), pp. 229-256.

[21] Sargent, T. (1998), "Discussion of Ball," Mimeo, Stanford University.

[22] Srour, G. (1999), "Inflation Targeting under Uncertainty," Bank of Canada Technical Report No. 85.

[23] Svensson, L. E. O. (1999), "Monetary Policy Issues for the Eurosystem," *Carnegie-Rochester Conference Series on Public Policy*, 51-1, pp. 79-136.

[24] ——— (2000), "Open-Economy Inflation Targeting," *Journal of International Economics* 50(1), pp. 155-184.

[25] Söderström, U. (1999), "Should Central Banks Be More Aggressive?" Sveriges Riksbank Working Paper Series No. 84.

[26] ——— (2002), "Monetary Policy with Uncertain Parameters," *Scandinavian Journal of Economics 104 (1),* pp. 125-145.

[27] Taylor, J. (1993), "Discretion Versus Policy Rules in Practice," *Carnegie-Rochester Conference Series on Public Policy*, 39, pp. 195-214.

[28] ——— (1999), "Robustness and Efficiency of Monetary Policy Rules as Guidelines for Interest Setting by the European Central Bank," *Journal of Monetary Economics*, 43, pp. 655-679.

[29] ——— (2001), "The Role of the Exchange Rate in Monetary Policy Rules," *American Economic Review: Papers and Proceedings* 91, May, pp. 263-267.

[30] Wieland, V. (1998), "Monetary Policy and Uncertainty About the Natural Unemployment Rate," Finance and Economics Discussion Paper No. 1998-22, Board of Governors of the Federal Reserve System.

[31] Woodford, M. (1999), "Optimal Monetary Policy Inertia," NBER Working Paper No. 7261.

[32] ——— (2001), "The Taylor Rule and Optimal Monetary Policy," *American Economic Review: Papers and Proceedings* 91, May, pp. 232-237.